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NUMERICAL DETERMINATION OF AERODYNAMIC CHARACTERISTICS OF CERTAIN WINGS OF FINITE DIMENSIONS

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We are going to present here a method of computing the aerodynamic characteristics of a particular type of wing of finite span under the conditinns of a streamlined, gaseous flow.

If we disregard the problem of the interference between the wing and the fuselage, these wings can be computed to within any degree of precision by using the method here offered.

No examples of the computation of such wings of finite span can be found in the contemporary literature on gas dynamics.

1. Shapes of Wings Examined

Let us suppose that the airfoil ABCDA in the drawing inserted represents the cross section of the wing at the points of contact with the fuselage, while one airfoil corresponds to the profile of the same wing in its consolic cross section.

We shall examine such wings ACHF, whose upper and lower surfaces combine to form the side surface of a truncated cone with the bases ABCDA and FGHKF and the apex at E. And we shall assume that the

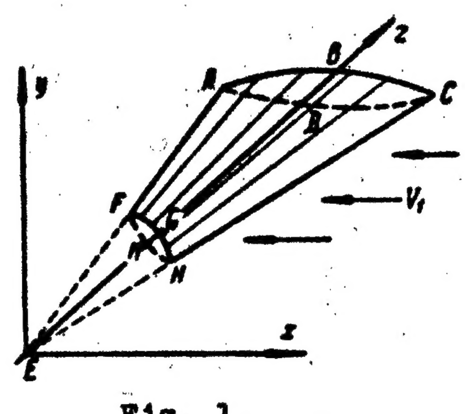

Fig. 1.

consolic cross section was chosen in a manner which either eliminated, or minimized to within a neghigible margin, the effect of the consolic cross section. Specifically, this will always be true if the consolic cross section degenerates into the geometric apex E of the wing.

2. Differential Equations and Boundary Conditions.

The particular shape of the wings examined here immediately suggests that the problem of streamlined flow should be considered in a manner reflecting the special wing-shape.

For, certainly, all points of any straight line from the apex E of this cone, whose side surface is the surface of the wing, are going to be in identical position relative to the wing's surface and the oncoming flow.

It is natural, therefore, to expect that under streamlining the hydrodynamic elements shall be constant along each such line.

Furthermore, it is natural to expect that the resulting surfaces of strong disruption of the stream shall also be conical surfaces with apex at the same point E of the streamlined-fluxed cone of the wing.

To verify the validity of these expectations, we shall pass from the rectangular coordinates x, y, z to different coordinates. The origin of that system will coincide with E, but we shall introduce two new coordinates ξ and η according to formulas

$$\xi = \frac{x}{z}, \quad \eta = \frac{y}{z}. \tag{2,1}$$

Let us now consider the system of differential equations of motion, and transform to the coordinates ξ and η with the assumption that the gas-dynamic elements depend exclusively on these coordinates.

Thus, we obtain the following system of differential equations:

$$(v_x - v_z\xi)\frac{\partial v_x}{\partial \xi} + (v_y - v_z\eta)\frac{\partial v_x}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi}, \tag{2,2}$$

$$(v_x - v_z\xi)\frac{\partial v_y}{\partial \xi} + (v_y - v_z\eta)\frac{\partial v_y}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \eta}, \tag{2,3}$$

$$(v_x - v_z\xi)\frac{\partial v_z}{\partial \xi} + (v_y - v_z\eta)\frac{\partial v_z}{\partial \eta} = \frac{1}{\rho}\left(\xi\frac{\partial p}{\partial \xi} + \eta\frac{\partial p}{\partial \eta}\right), \tag{2,4}$$

$$(v_x - v_z\xi)\frac{\partial}{\partial \xi}\lg\rho + (v_y - v_z\eta)\frac{\partial}{\partial \eta}\lg\rho + \frac{\partial v_x}{\partial \xi} + \frac{\partial v_y}{\partial \eta} - \xi\frac{\partial v_z}{\partial \xi} - \eta\frac{\partial v_z}{\partial \eta} = 0, \tag{2,5}$$

$$(v_x - v_z\xi)\frac{\partial}{\partial \xi}\left(\frac{p}{\rho^k}\right) + (v_y - \eta v_z)\frac{\partial}{\partial \eta}\left(\frac{p}{\rho^k}\right) = 0. \tag{2,6}$$

Since our system contains none of the x, y, z coordinates, but only their combinations in terms of the magnitudes ξ and η, we can seek solutions depending only on ξ and η. This is sufficient to justify our expectations, since it is obvious that with the actual conicity of the streamlined surfaces and the assumed conicity of the surfaces of strong disruption in the gaseous flux we shall be able to express all boundary conditions in the terms of these new coordinates ξ and η.

Let the equation ofthe streamlined cone be:

$$\frac{y}{z} = \Phi\left(\frac{x}{z}\right). \tag{2,7}$$

On computing the direction cosines of the normal to the surface of the cone with coordinate axes x, y, z, and on equating the normal component of the wing's velocity to zero, we shall obtain the streamlining condition in the form:

$$v_y - \eta v_z = (v_x - \xi v_z)\frac{d\Phi(\xi)}{d\xi}. \tag{2,8}$$

The condition (2,8) is expressed in terms of ξ and η, which justifies our expectations.

We shall pass now to the conditions on the surface of the strong disruption. Let

$$\frac{y}{z} = X\left(\frac{x}{z}\right) \tag{2,9}$$

be the equation of this surface.

The x axis will be directed in the direction of the flow. We shall compute the direction cosines of the normal to that surface and substitute these in the expressions holding for disruption surfaces. The relation on the surface of strong disruption will have the forms:

$$v_{x,2} = V_1 - \frac{2}{k+1}\frac{1}{V_1}\left[V_1^2\frac{X'^2}{1+X'^2+(\xi X'-\eta)^2} - a_1^2\right] \tag{2,10}$$

$$v_{y,2} = \frac{2}{k+1}\frac{1}{V_1}\frac{1}{X'}\left[V_1^2\frac{X'^2}{1+X'^2+(\xi X'-\eta)^2} - a_1^2\right], \tag{2,11}$$

$$v_{z,2} = \pm\frac{2}{k+1}\frac{1}{V_1}\frac{\eta-\xi X'}{X'}\left[V_1^2\frac{X'^2}{1+X'^2+(\xi X'-\eta)^2} - a_1^2\right], \tag{2,12*}$$

$$p_2 = \frac{2}{k+1}\rho_1\left[V_1^2\frac{X'^2}{1+X'^2+(\xi X'-\eta)^2} - \frac{k-1}{2k}a_1^2\right], \tag{2,13}$$

$$\rho_2 = \frac{\rho_1}{\frac{k-1}{k+1}+\frac{2}{k+1}\left(\frac{a_1}{V_1}\right)^2\frac{1+X'^2+(\xi X'-\eta)^2}{X'^2}}, \tag{2,14}$$

*The sign in formula (2,12) will depend on the choice of the direction of the axis.

The index 2 in these relations corresponds to the hydrodynamic elements after the disruption, and the index 1 to the values prior to the disruption.

These relations (2,10) - (2,14) are again expressed in the terms ξ and η ; consequently, our expectations are fully justified.

In reference to all the above facts, we conclude that our investigations must be applied in the plane of these two coordinates (ξ, η). In this plane we shall have to integrate the system of equations (2,2) to (2,6) with the boundary condition (2,8) on the profile and with the boundary conditions (2,10) to (2,14) on the still unknown line of disruption.

Should bhe system of the equations (2,2) to (2,6) turn out to be hyperbolic, then the problem can be solved by applying the characteristic method in the numerical integration of hyperbolic systems.

3. The Type of the System of Equations and Their Characteristics

If the equation

$$\eta = \eta(\xi) \tag{3,1}$$

is the characteristic equation of the system (2,2) - (2,6), the usual methods will lead to the following equation for the determination of the characteristic directions (characterized by angular coefficient $\frac{d\eta}{d\xi}$):

$$\begin{vmatrix} A, & 0, & 0, & -\frac{1}{\rho}\frac{d\eta}{d\xi}, & 0 \\ 0, & A, & 0, & \frac{1}{\rho}, & 0 \\ 0, & 0, & A, & \frac{1}{\rho}\left(\xi\frac{d\eta}{d\xi}-\eta\right), & 0 \\ -\frac{d\eta}{d\xi}, & 1, & \xi\frac{d\eta}{d\xi}-\eta, & 0, & \frac{1}{\rho}A \\ 0, & 0, & 0, & A, & -a^2A \end{vmatrix} = 0, \tag{3,2}$$

where

$$A = -\frac{d\eta}{d\xi}(v_x - \xi v_z) + (v_y - \eta v_z)$$

and a is the velocity of the sound.

Equation (3,2) is of the fifth degree in $\frac{d\eta}{d\xi}$. In solving it we shall obtain five roots.

On expanding the determinant (3,2) and solving the

sulting equation, we find that it has only three distinct roots, because one of these roots occurs three times.

For the roots of the characteristic determinant, we obtain the expressions:

$$\left(\frac{d\eta}{d\xi}\right)_{1,2} = \frac{(v_x - v_z\xi)(v_y - v_z\eta) - a^2\xi\eta}{(v_x - v_z\xi)^2 - a^2(1 + \xi^2 + \eta^2)} \pm$$

$$\pm \frac{a\sqrt{(v_x - v_z\xi)^2 + (v_y - v_z\eta)^2 + (\eta v_x - \xi v_y)^2 - a^2(1 + \xi^2 + \eta^2)}}{(v_x - v_z\xi)^2 - a^2(1 + \xi^2 + \eta^2)}, \quad (3,3)$$

$$\left(\frac{d\eta}{d\xi}\right)_{3,4,5} = \frac{v_y - v_z\eta}{v_x - v_z\xi}. \quad (3,4)$$

The roots of the characteristic determinant $\left(\frac{d\eta}{d\xi}\right)_{3,4,5}$ are always real, while the roots of $\left(\frac{d\eta}{d\xi}\right)_{1,2}$ will be real if the inequality

$$\frac{(v_x - \xi v_z)^2 + (v_y - \eta v_z)^2 + (v_x\eta - v_y\xi)^2}{1 + \xi^2 + \eta^2} > a^2. \quad (3,5)$$

holds. In that case, the system (2,2) - (2,6) will be perfectly hyperbolic and the characteristic method can be applied.

To obtain an idea as to the possible regions of complete hyperbolicity for the system (2,2) - (2,6), we shall consider a flow having constant hydrodynamic elements and moving along the x axis with the supersonic velocity V_1 while the velocity of sound is a_1.

In this case, the inequality (3,5) shall be transformed into the inequality

$$\left(\frac{\xi}{\frac{\sqrt{V_1^2 - a_1^2}}{a_1}}\right)^2 - \eta^2 < 1. \quad (3,6)$$

This inequality holds in the region between the two branches of the hyperbola. The region of perfect hyperbolicity will expand as Mach's number increases. When Mach's number is less than 1, this region vanishes.

4. Relation on the Characteristics and Application of the Characteristic Method

The relations on the characteristics are obtained in the usual way. The roots of the characteristic equation are substituted sequentially into the characteristic system, containing partial derivatives of the unknown functions with respect to one independent variable. The resulting relations do not determine all partial derivatives, but the latter can be disregarded. As a result of the exclusion of these partial derivatives, we obtain the relations on the characteristics.

There will be five independent relations.

We will get one relation by excluding the partial derivatives from the formulas which we obtained by substituting

the roots $(\frac{d\zeta}{d\xi})$ in the characteristic system; and three formulas by substituting the roots $(\frac{d\zeta}{d\xi})_{3,4,5}$ in the same system.

As a result of the computations we get the following relations on the characteristics:

$$(v_y - \eta v_z)\,dv_x - (v_x - \xi v_z)\,dv_y + (\eta v_x - \xi v_y)\,dv_z =$$
$$= \frac{1}{a^2}\left\{\frac{d\eta}{d\xi}\left[(v_x - \xi v_z)^2 - a^2(1+\xi^2) - (v_y - \eta v_z)(v_x - \xi v_z)\right] + a^2\xi\eta\right\}\frac{dp}{\rho}, \quad (4,1)$$

$$(v_x - \xi v_z)\,dv_x + (v_y - \eta v_z)\,dv_y = -\frac{dp}{\rho}, \quad (4,2)$$

$$(\eta v_x - \xi v_y)\,dv_y + (v_x - \xi v_z)\,dv_z = \xi\frac{dp}{\rho}, \quad (4,3)$$

$$dp = a^2 d\rho. \quad (4,4)$$

= The relation (4,1) holds along the characteristics with the inclinatinn $\frac{d\zeta}{d\xi} = (\frac{d\zeta}{d\xi})_{1,2}$. Formulas (4,2) to (4,4) hold on characteristics with the inclination $\frac{d\zeta}{d\xi} = (\frac{d\zeta}{d\xi})_{3,4,5}$.

Given the five relations on the characteristics and the boundary conditions, we can replace these by formulas in finite differences and thus find the five unknown functions v_x, v_y, v_z, P and p.

The order in which these functions will be determined will correspond, in general, to the order in which solutions of the planar whirlwind problem of gas dynamics are found. Accordingly, see any adequate textbook on gas dynamics.

No difficulties in principle will be encountered; thus, there is no need to describe in detail the application of these operations to a practical problem.

5. Wings of Low Conicity and a Slight Arrow-Form.

If the wing in question has a low conicity, and is only slightly arrow-shaped, the values of ξ and η in the velocity regions affecting the values of the hydrodynamic elements upon the surface of the wing shall be small.

Let us assume that in these regions the values of ξ and η are such that their squares can be disregarded compared to unity; let us consider our problem under this conditon.

It is clear that for such wings in these regions where the values of the hydrodynamic elements depend on the wing, the order of the magnitude v_z and of its partial derivatives will be the same as the order of the magnitudes $|V_1\xi|$ and $|V_1\eta|$, where V_1 is the velocity of unperturbed flow.

Beeause of this conditon, we can consider the magnitude $v_x - \xi v_z$ as equal to v_x* to within the accuracy assumed in the discussion. Simultaneously, $\xi\frac{\partial v_z}{\partial \xi}$ and $\eta\frac{\partial v_z}{\partial \eta}$ will be negligible in comparison with magnitudes of the order of V_1.

Taking the above into consideration, we shall get

*and the magnitude $v_y - \eta v_z$ as equal to v_y.

instead of the system (2,2) - (2,6), the system:

$$v_x \frac{\partial v_x}{\partial \xi} + v_y \frac{\partial v_x}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi}, \tag{5,1}$$

$$v_x \frac{\partial v_y}{\partial \xi} + v_y \frac{\partial v_y}{\partial \eta} = -\frac{1}{\rho}\frac{\partial p}{\partial \eta}, \tag{5,2}$$

$$v_x \frac{\partial v_z}{\partial \xi} + v_y \frac{\partial v_z}{\partial \eta} = \frac{1}{\rho}\left(\xi \frac{\partial p}{\partial \xi} + \eta \frac{\partial p}{\partial \eta}\right), \tag{5,3}$$

$$v_x \frac{\partial}{\partial \xi} \lg \rho + v_y \frac{\partial}{\partial \eta} \lg \rho + \frac{\partial v_x}{\partial \xi} + \frac{\partial v_y}{\partial \eta} = 0, \tag{5,4}$$

$$v_x \frac{\partial}{\partial \xi}\left(\frac{p}{\rho^k}\right) + v_y \frac{\partial}{\partial \eta}\left(\frac{p}{\rho^k}\right) = 0. \tag{5,5}$$

Equations (5,1), (5,2), (5,4), and (5,5), considered as a whole, are the same as a system of equations for the planar motion of a gas. And as soon as these equations are integrated, the integration of (5,3) can be performed quite easily.

The streamlining condition (2,8) is the same in this case as the streamlining condition in a planar flow problem, and has the form

$$v_y = \Phi'(\xi) v_x. \tag{5,6}$$

And, finally, the boundary conditions (2,10) to (2,14) assume, under the assumptions made, the following forms:

$$v_{x,2} = V_1 - \frac{2}{k+1}\frac{1}{V_1}\left[V_1^2 \frac{X'^2}{1+X'^2} - a_1^2\right], \tag{5,7}$$

$$v_{y,2} = \frac{2}{k+1}\frac{1}{V_1 X'}\left[V_1^2 \frac{X'^2}{1+X'^2} - a_1^2\right], \tag{5,8}$$

$$v_{z,2} = \pm \frac{2}{k+1}\frac{1}{V_1}\frac{\eta - \xi X'}{X'}\left[V_1^2 \frac{X'^2}{1+X'^2} - a_1^2\right], \tag{5,9}$$

$$p_2 = \frac{2}{k+1}\rho_1\left[V_1^2 \frac{X'^2}{1+X'^2} - \frac{k-1}{2k} a_1^2\right], \tag{5,10}$$

$$\rho_2 = \frac{\rho_1}{\frac{k-1}{k+1} + \frac{2}{k+1}\left(\frac{a_1}{V_1}\right)^2 \frac{1+X'^2}{X'^2}}. \tag{5,11}$$

The conditions (5,7), (5,8), (5,10) and (5,11) are the same as the corresponding boundary conditions of a planar problem.

Condition (5,9) must be used to determine the quantity v_z. We obtain the quantity v_z by integrating the equation (5,3), using the boundary condition obtained from condition (5,9).

Thus, we conclude that, in order to solve the problem of streamlining for a wing of low conicity and a slight arrow-form, it suffices to solve the problem of streamlining of a wing of an infinite span having a profile $\eta = \Phi(\xi)$.

Then, we transform from the variables ξ and η to the variables x, y, z. Thus, the computation of the aerodynamical characteristics of the wings under consideration differs only negligibly from the computation of the aerodynamic characteristics for wings of infinite span.

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